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POSSIBLE MECHANISMS FOR THE HUBBLE-SANDAGE (S DORADUS) VARIABLES

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ABSTRACT

The brightest nonexplosive stars known at present are the Hubble-Sandage (or S Doradus) variables. It is generally assumed that these objects consist of intrinsically luminous, massive stars embedded in clouds of circumstellar gas and dust. The unexplained long-term variability of their light is investigated in the present paper in terms of the following possible mechanisms: (1) episodic, nearly catastrophic mass loss, perhaps accompanied by temporary shrinkage of the stellar radius, in a luminous supergiant; (2) flickering of a hydrogen-burning or helium-burning shell; (3) hydrogen flashing in an evolved stellar core; (4) pulsation of a luminous supergiant envelope near the Eddington limit of radiative stability; (5) pulsation of a dense circumstellar gas or dust cloud; (6) overturning of giant convection cells (or some other kind of nonradial oscillation) in a luminous supergiant envelope; or (7) vibrational instability and mass outflow in an extremely massive main-sequence star. Among these suggestions, the second and third can be almost definitely ruled out, while the sixth and seventh ones are also rather unlikely.

Preference is given here to the notion that a sudden, massive outflow of matter may sometimes occur from the surface of an evolved supergiant of very high mass. Model calculations indicate that such a star can temporarily become quite blue and visually much fainter, although the bolometric light output of the star remains nearly constant. If the ejected cloud of matter is optically thick, the object as a whole may appear redder and hence also visually fainter (as Davidson originally suggested). In both cases, the observed sharp declines in blue light and very slow recoveries can be adequately accounted for. To explain the peculiar light oscillations with "periods" of 3–10 yr, we suggest that, if they are not simply due to rapidly recurring mass outflows, then either the stellar envelope (if the luminosity lies close to the Eddington limit) or the circumstellar cloud (if it is optically thick enough) may be undergoing bulk radial pulsations in the fundamental mode. Perhaps the outward mass flux from the star (or whatever causes this flux) drives the pulsations.

Subject headings: stars: circumstellar shells — stars: interiors — stars: pulsation — stars: supergiants — stars: variables

I. INTRODUCTION

The Hubble-Sandage variables constitute a unique class of very luminous, blue, irregular variable stars, recognized originally in external galaxies (Hubble 1926, 1929; Hubble and Sandage 1953; Tammann and Sandage 1968; Rosino and Bianchini 1973; Sharov 1973). The light variations of these rare objects exhibit at least three different kinds of behavior: (1) Rapid, erratic fluctuations of (usually) small amplitude; (2) slow quasi-periodic (3–10 yr) oscillations of significantly greater amplitude (up to 2 mag in blue light); and (3) precipitous declines (greater than 3 mag in blue light) within as little as 1 yr, followed by very slow recoveries lasting perhaps many decades. Not all the members of this group have shown all three scales of light variation; however, the available photometric histories are not continuous and cover less than a century. Hubble and Sandage (1953), in their classic study, mentioned S Doradus of the Large Magellanic Cloud as a possible member of the group, and this star is now often designated as the group prototype (e.g., Kukarkin *et al.* 1974). Its light curve (Gaposchkin 1943; van Genderen 1979), as well as the light curve of η Carinae in our Galaxy (Feinstein and Marraco 1974), resembles in many respects the light curves of the classical Hubble-Sandage variables, with which other very luminous variables have also been compared (e.g., Thackeray 1974; Sharov 1975; Gottlieb and Liller 1978; van Genderen 1979). Spectra and colors, though not available in large number, suggest that probably all of these stars lie embedded in circumstellar material which they are ejecting now or have ejected in the recent past. However, it is not clear how much of the observed variability is due to physical changes in the circumstellar material and how much is intrinsic to the stars themselves. The true photosphere of the star is probably, in many cases, not directly visible, since the circumstellar gas and dust act to convert the star's high-energy radiation to longer wavelengths.

Most of the theoretical speculation about these objects has focused on the best-observed representative, η Car. Traditionally, η Car has been regarded as a slow nova or a slow supernova (e.g., Payne-Gaposchkin 1957; Zwicky 1965). Ostriker and Gunn (1971) (see also Davidson and Ostriker 1972) have proposed that, if indeed a supernova, η Car may be currently powered by a central pulsar, although subsequent observations have uncovered no obvious short-period oscillations (Lasker and Hesser 1972) and no obvious nonthermal radiation, even in the far-infrared (Gehrz *et al.* 1973; Harvey, Hoffmann, and Campbell 1978) or X-ray region (Seward *et al.* 1979). Gratton (1963) suggested that η Car may be, rather, a newly formed massive star, rapidly contracting toward the main sequence. However, Burbidge (1962) and several other authors (Burbidge and Stein 1970; Talbot 1971; Davidson 1971; Hoyle, Solomon, and Woolf 1973; Humphreys and Davidson 1979) have given reasons for preferring an extremely massive star now in (or beyond) the main-sequence phase and losing (or having lost) a considerable portion of its initial mass as a result of surface shock waves arising from violent nuclear-energized pulsations excited in the stellar core. More recently, Bath (1979) has suggested that η Car may be a more conventionally massive main-sequence star which is rapidly accreting matter from an unseen binary companion and is thereby acquiring a temporary overluminosity generated by gravitational potential energy release. Tutukov and Yungelson (1979) have stressed that what is actually seen in such a situation may be only the common envelope surrounding the two components of the system. Or perhaps a rotating ring can better explain the observed annular structure of η Car's envelope (Hyland *et al.* 1979; Warren-Smith *et al.* 1979). On the other hand, Andriesse, Packet, and de Loore (1981) and Davidson, Walborn, and Gull (1982) have returned to the idea of an extremely massive, evolved single supergiant, which they suggest is losing mass in the form of an enormous stellar wind.

Interesting though these suggestions may be, none of them has thus far progressed very much beyond the speculative stage in dealing with the structure and evolution of the underlying star, and none has really addressed the question of the observed long time scales of variability. In fact, it is not known for certain whether such an extraordinarily luminous object as η Car is a typical member of the class of Hubble-Sandage variables, which may not necessarily comprise a homogeneous group (Sandage and Tammann 1974). Yet, except for Bath's (1979) suggestion, the general consensus concerning the traditional members points to a very massive hot star rapidly ejecting matter in a pre–main-sequence stage of evolution (Martini 1969) or in some post–main-sequence stage (Sterken and Wolf 1978; Humphreys and Davidson 1979; Wolf, Appenzeller, and Cassatella 1980; Gallagher, Kenyon, and Hege 1981). A pre–main-sequence stage is thought to be less likely by Gallagher, Kenyon, and Hege (1981) on the grounds that some Hubble-Sandage variables are found outside dusty regions and show little circumstellar H II. Bath's (1979) binary hypothesis, we believe, has difficulty in explaining why Hubble-Sandage variables are found only at very high luminosities and why no definite evidence of binary membership has ever been obtained. An early attempt to explain the major light decreases of S Dor as eclipses (Gaposchkin 1943) had ultimately to be abandoned (Wesselink 1956; Thackeray 1974).

For the foregoing reasons, we have embarked on an investigation of the possible physical instabilities that may arise in very massive single stars evolving between the zero-age main sequence and the presupernova state. Our goal is to try to account for the observed periods and amplitudes of the slower variations shown by the Hubble-Sandage variables. The fundamental types of instabilities that are involved here are secular and pulsational. Although seven specific mechanisms will be examined, we state here briefly our main conclusions: (1) The Hubble-Sandage variables are probably intrinsically luminous, intrinsically blue, post–main-sequence stars of extremely high mass which possibly lie near the Eddington limit of radiative stability and which are experiencing recurrent episodes of nearly catastrophic mass loss. (2) The exact source of instability in these variables has not been determined but is probably dynamical and/or pulsational in character and probably resides in the outer layers of their envelopes or, possibly, in their dense circumstellar shells.

II. THE BASIC STELLAR MODEL

To represent a typical Hubble-Sandage variable with $L/L_{\odot} \approx 10^6$, we have rather arbitrarily chosen an initial stellar mass of 60 $M_{\odot}$ and initial chemical composition parameters $(X, Z) = (0.739, 0.021)$. Radiative opacities have been taken to be the standard Cox-Stewart values. The effects of convection, semiconvection, and mass loss have been modeled numerically as in our earlier work on massive stars (Stothers and Chin 1979), but several computational algorithms have been upgraded to handle accurately mass loss rates of as much as 1 $M_{\odot}$ yr^{-1} and time steps as small as 10^{-1} yr for the supergiant models.

Rotation and magnetic fields have been ignored in the present study, although it should be pointed out that they may give rise to additional kinds of instability.

To avoid possible confusion in astronomical terminology, we shall refer to the two main divisions of the stellar interior as *core* and *envelope*. The ejected circumstellar material will be said to form a *cloud*, or *shell* (the prevailing context ought to prevent confusion with a nuclear-burning shell inside the star).

III. SECULAR INSTABILITIES

a) *Surface Mass Loss*

A large number of theoretical studies have convincingly shown that models of extremely massive stars, evolving without mass loss, deplete their core helium as yellow or red supergiants (Stothers and Chin 1968, 1976, 1979, 1981; Ziółkowski 1972; Varshavsky and Tutukov 1973; Chiosi, Nasi, and Sreenivasan 1978; Massevitch *et al.* 1979; Maeder 1981). Observations, however, have failed to turn up any of the predicted supergiants. One possible explanation for this discrepancy, as noted by many authors, is that exceptionally rapid mass loss sets in shortly after the star leaves the main sequence. If the ejection of mass is of a sporadic nature, then perhaps an accompanying light variation may be produced at an observable level. Even if mass loss is strictly continuous, long-term light oscillations may develop as a consequence of thermal readjustments of the envelope. Or perhaps something entirely unforeseen may occur. Here we examine the consequences of assuming very rapid mass loss from a post–main-sequence star of initially 60 $M_\odot$.

In order to avoid the possibility of any confusion with effects from thermal instability in the hydrogen-burning shell, we shall artificially suppress convective mixing in the immediate vicinity of the shell (§ III*b*). We shall then find that the burning shell remains permanently thermally stable. Another consequence will be that the star is able to evolve quickly and immediately into the dimensions of a red supergiant while the helium core is still contracting (Stothers and Chin 1968). We further assume that, once the star has moved significantly away from the main sequence, mass is suddenly ejected at a large, constant rate, taken to be 10^{-2} $M_\odot$ yr^{-1}. Our results show, surprisingly, that the star continues to move across the H-R diagram on a normal envelope Helmholtz-Kelvin time scale,

$$\tau_{\rm HK} \sim GM^2/(RL),$$

in consequence of which the star reaches the region of red supergiants in about 1800 yr with a mass of 42 $M_\odot$. There is no indication at all of secular instability in the envelope other than the normal process of envelope expansion, even though we have used very small time steps between successive models. Moreover, further experiments have shown that, if mass loss is suddenly shut off or suddenly reinitiated, the results turn out to be essentially the same.

To obtain a larger effect, a greater rate of mass loss must be employed. Let us suppose that the rate of mass loss becomes nearly catastrophic at some critical effective temperature $T_{\rm crit}$. This situation could arise, for example, if dynamical instability suddenly occurred in the stellar envelope as a result of any of the following processes (which become more likely as the effective temperature is lowered): (1) a stellar-luminosity excess over the Eddington limit (§ IV*a*); (2) a significant density inversion in the stellar envelope (Peterson 1971; Bisnovatyi-Kogan and Nadyoshin 1972; Schmid-Burgk and Scholz 1975; but see Underhill 1949; Bisnovatyi-Kogan 1973); (3) random instabilities associated with turbulent convection in a low-gravity environment (Fusi-Pecci and Renzini 1975; Andriesse 1980); (4) widespread partial ionization of hydrogen and helium throughout the envelope (Paczyński and Ziółkowski 1968; Stothers 1972); or (5) Roche-lobe overflow due to the presence of a binary companion (Warren-Smith *et al.* 1979). To take a concrete example, we consider a stellar model that has evolved, without prior mass loss, to an arbitrary $T_{\rm crit} = 5000$ K, whereupon mass is assumed to flow away at a specified rate of 0.3 $M_\odot$ yr^{-1}. We may retain the usual approximation of hydrostatic equilibrium in the models because none of the time steps employed turn out to be smaller than the fundamental period of radial pulsation. Furthermore, since the basic driving mechanism (whatever it may be) is almost certain to be subphotospheric, we may assume that the outflow continues for some years (say, a suitable fraction of an envelope Helmholtz-Kelvin time) after the effective temperature begins to shift back significantly toward values higher than $T_{\rm crit}$.

Results of our calculations are shown on the H-R diagram in Figure 1, where the two tracks presented were computed with slightly different assumptions about the choice of time step. Notice that, during the initial stages of mass ejection, the surface luminosity drops by about 0.05 mag but then quickly recovers to its original level. After the star's mass has fallen to 46 $M_\odot$, there is a violent blueward shift ($dT_e/dt \approx 1000$ K yr^{-1} and $dR/dt \approx -0.6$ km s^{-1}), which causes the star's *blue magnitude* to fade at a rate of about 0.2 mag yr^{-1} owing to the effect of bolometric correction. If mass loss is suddenly turned off, however, the blueward motion is quickly halted. At the same time, the star's luminosity abruptly drops by about 0.09 mag. Thereafter, the luminosity begins gradually to rise again, and the surface cools off at a much slower rate of about 50 K yr^{-1} (which is the normal rate for this stage of evolution). The quick resumption of radius expansion proves that the rapid blueward shift does not belong in the class of ordinary "blue loops" which appear in standard evolutionary calculations and which can also be induced by mass loss (though at more moderate rates of mass ejection). Since the star's natural tendency in the present instance is to expand, it will undergo mass loss again if its effective temperature regains the critical value $T_{\rm crit}$. In this way, recurrent episodes of mass ejection could well occur. Further calculations support this conjecture; they also indicate that, when the core is burning helium,

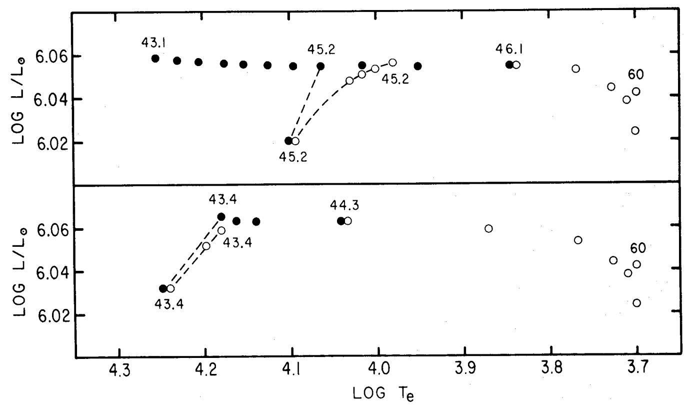

FIG. 1.—Theoretical H-R diagram showing two evolutionary tracks with mass loss for a star of initially 60 $M_\odot$. Mass loss begins when $\log T_e = 3.70$ (the preceding evolutionary history is not shown) and continues at a rate of 0.3 $M_\odot$ yr^{-1}. Intervals of time elapsed between successive open circles are 10 yr, and between successive filled circles, 1 yr. Masses are indicated in solar units. Dashed lines refer to the star's evolution after mass loss is abruptly turned off.

reexpansion of the radius occurs at a rate $\sim 10^2$ times slower than when the core is rapidly contracting.

Predicted B-magnitude light curves for the two evolutionary tracks just discussed are shown in Figure 2; bolometric corrections and $B - V$ colors as a function of effective temperature have been taken from Buser and Kurucz (1978). For a number of reasons, our results must be regarded as merely illustrative. For one thing, our choice of T_{crit} is arbitrary since its true value is not known. If T_{crit} had been chosen to be larger than 5000 K (observations suggest that it is), the rising branch of the B light curve would be less steep. Furthermore, the intervals of time between episodes of mass loss would be shorter. As a particular example, consider the choice $T_{\text{crit}} = 11{,}000$ K, and let the mass loss cease after 1 yr. Suppose that previous episodes of mass loss have reduced the star's mass to 45.5 $M_\odot$. Figure 1 then suggests that the star's B magnitude would at first fade by 0.5 mag in about 2 yr but would brighten again over the next 10 yr, after which the cycle would repeat itself. Although T_{crit} may actually change somewhat from episode to episode, it probably varies much more strongly from star to star, depending in an unknown way on stellar mass, luminosity, chemical composition, etc. A second reason for regarding our results as only illustrative is that we do not know how suddenly mass loss actually turns off. Third, the speed with which the stellar radius contracts as a result of mass loss, and hence the abruptness with which the B light curve falls, is critically dependent on the chosen rate of mass loss. We have seen that, to have the radius contract at all, the rate has to exceed 10^{-2} $M_\odot$ yr^{-1}. But this value must be strongly model dependent since it hinges on, among other things, the amount of mass already lost: a relatively less massive envelope does not need as large a rate of mass loss to undergo sudden radius contraction. Nonetheless, Figures 1 and 2 are expected to give a qualitatively valid picture of what happens to the star under the envisaged circumstances.

Davidson (1971) has suggested, in a discussion of η Car, that the matter coming off the star may be optically thick, especially if grains condense from the gas as it cools. In this case, the star may not be seen directly, or else may be seen only faintly. The whole object will therefore appear redder with time and, because of the effect of bolometric correction, dimmer in blue light as well. When the ejected cloud has dispersed sufficiently to become optically thin, the underlying hot star will appear again, even though it may be past its bluest stage. The predicted B-magnitude light curve, in this case, would be not unlike the curves shown in Figure 2; however, the associated $B - V$ color at minimum light would be red, not blue. There now arises the curious

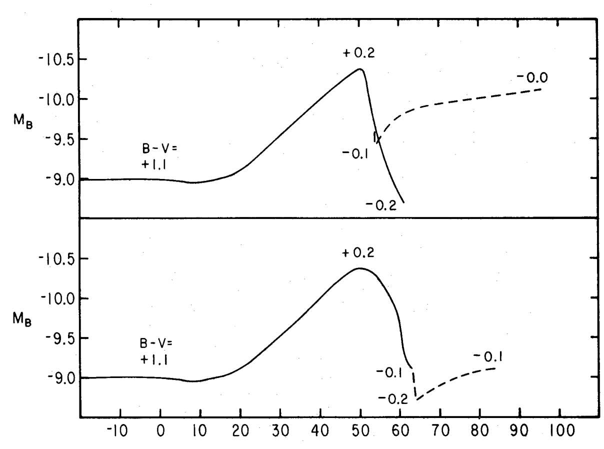

FIG. 2.—Theoretical B-magnitude light curves for an evolved star of initially 60 $M_\odot$ which is undergoing rapid mass loss. The curves refer to the evolutionary tracks shown in Fig. 1. Predicted $B - V$ colors are indicated at several stages.

possibility that the reddening produced by the ejected material might just cancel the blueward color shift of the star itself, so that the composite object would show no significant light or color variation!

As another possibility, based in part on Martini's (1969) discussion of S Dor, we suppose that the star is initially very hot in its quiescent state but is surrounded by a semitransparent cloud ejected during the previous outburst, so that the object seen as a whole emits mostly at ordinary blue and visual wavelengths. As the cloud continues to disperse, the partially hidden, ultraviolet-bright star will emerge more fully into view, and therefore the object will appear dimmer in blue light. This picture predicts a slow decline in blue light, together with a fast rise when the next cloud of matter is ejected, unlike the curves displayed in Figure 2 (even though the $B-V$ color at maximum light would still be redder than at minimum light). If, however, because of an abrupt change of chemical phase, the expanding cloud *suddenly* became optically thin, the fading of the whole object in blue light would be very rapid. But, in that case, one would predict a long, flat preceding maximum, which, together with the fast rise, would again not resemble Figure 2.

Turning to the actual observations of Hubble-Sandage variables, we can divide the large-amplitude variables into two subgroups: (1) those stars that seem, according to their colors, to be only moderately obscured by circumstellar material, so that the observed light and color variations may be taken as reflecting essentially the variations of the stars themselves; and (2) the highly reddened variable stars that are surrounded by very optically thick clouds of dust.

Within the first subgroup, Variable 22 in NGC 2403 has a light history that shows a close resemblance to the theoretical light curves displayed in Figure 2. After a very slow increase of brightness that lasted more than 30 yr, the star's blue magnitude suddenly faded by ~ 1.5 mag in about 3 yr, while its $B-V$ color became bluer, at least in its later stages of fading, by 0.3 mag (Tammann and Sandage 1968). Another variable star of apparently the same type is the blue Variable 2 in M33 (Hubble and Sandage 1953; Rosino and Bianchini 1973; Humphreys 1975, 1978). Superficially at least, an almost identical behavior has been observed several times in S Dor, which drops by 0.1–2 mag in roughly 3 yr, at which time it is both bluer and of earlier spectral type (Gaposchkin 1943; Wesselink 1956; Martini 1969; van Genderen 1979). However, this star brightens again much too rapidly (in ~ 7 yr) to accord with our present models and also shows, at maximum light, too small a rate of mass loss, viz., 5×10^{-5} $M_\odot$ yr^{-1} (Stahl and Wolf 1982). Although its rate of mass loss at minimum light is not known, it is expected to be even smaller (Thackeray 1974; Wolf, Appenzeller, and Stahl 1981). Nevertheless, we have given theoretical arguments above, based on a subset of our models, that suggest that S Dor's behavior might be reproduced by assuming a very brief (and hence difficult to observe) burst of very heavy mass loss, coupled with $T_{\rm crit} \approx 11{,}000$ K, which would accord better with S Dor's blue color at maximum light. Although a lower rate of mass loss, taken in conjunction with a thinner hydrogen envelope, could also explain the star's sudden decline in blue light, the recovery to maximum light would be much too slow (requiring more than 10^2 yr) according to our models with very thin envelopes; even S Dor's longer cycle of ~ 40 yr could not be explained with such models.

The second subgroup of Hubble-Sandage variables showing large amplitudes consists of Variable A in M33 (Hubble and Sandage 1953; Rosino and Bianchini 1973; Humphreys and Warner 1978), Variable 12 in NGC 2403 (Tammann and Sandage 1968), and η Car (Feinstein and Marraco 1974; Harvey, Hoffmann, and Campbell 1978). All these objects are known to have suffered a sudden, precipitous decline in blue light of more than 3 mag, which was accompanied by a very strong reddening of their colors. There exists fairly reliable evidence from the colors that the *bolometric* luminosities of these stars (indeed, of all the Hubble-Sandage variables) have remained approximately constant, which is certainly in accord with our theoretical predictions. It is also estimated that η Car has been ejecting mass at an average rate of 0.075 $M_\odot$ yr^{-1} (Andriesse, Donn, and Viotti 1978) or 0.022 $M_\odot$ yr^{-1} (Hyland *et al.* 1979) since the star's great outburst in 1843. Because the rate of mass loss at maximum must have been considerably higher (Humphreys and Davidson 1979), our rate of 0.3 $M_\odot$ yr^{-1}, which we employed as a typical example of rapid mass loss, may not be an unrealistic value. In the case of these three variable stars at least, such a high rate of mass loss seems to have produced a circumstellar cloud of very large optical thickness.

b) *Hydrogen-Shell Flickering*

It is important also to examine possible instabilities that may occur deep within the interior of a massive star. Following the exhaustion of hydrogen in the core, thermal (or secular) instability can arise in the surrounding hydrogen-burning shell under certain circumstances (Stothers and Chin 1972, 1976; Tanaka, Arimoto, and Takeuti 1981). The main requirements are that the burning region not be too thin in mass and that the hydrogen gradient through the shell be fairly steep. The latter condition presupposes the development of a fully convective zone (FCZ) immediately above the shell, so that the shell, while still thick, can burn into the hydrogen discontinuity that lies at the base of the FCZ.

Because convective-core overshooting, semiconvection, and surface mass loss greatly influence the shell's location in mass fraction (q_s) and its mean hydrogen gradient (dX/dq), we have reexamined the whole question of thermal instability in the shell by allowing greater flexibility in our choices of q_s and dX/dq. Standard models for stars of 60 $M_\odot$ show $q_s = 0.35$–0.38 and $dX/dq \geq 2$. We shall here consider, in addition, the choices $q_s = 0.26$ and $q_s = 0.69$, together with the full range of possible dX/dq values. In all cases, we find that thermal instability occurs if, and only if, $dX/dq \approx 10$–20. The instability appears as low-amplitude thermonuclear pulses that repeat themselves in cycles of hundreds to thousands of years. Relatively little of the pulse signal, however, reaches the stellar surface through the thick overlying layers of the envelope. Four consecutive cycles of our *most regular* pulse sequence are shown in Figure 3, while two consecutive cycles of our *largest amplitude* pulse sequence appear in Figure 4. Even in the most favorable case, the predicted light amplitude at the stellar surface does not exceed 0.02 mag, and, in this case, the cycle time between pulses is ~ 1500 yr.

Clearly, these hydrogen-shell pulses cannot account for the Hubble-Sandage variables. Helium-shell pulses, which occur later in the evolution, have been shown to be even weaker (Stothers and Chin 1973), and the more energetic and more rapid carbon-shell pulses (Sugimoto 1970; Ikeuchi *et al.* 1971) occur much too late in the evolution to be observable, since the remaining evolution time at such an advanced stage is only ~ 5 yr. More time would be available in stars of lower mass (Woosley, Weaver, and Taam 1980), but the surface luminosity would be too low.

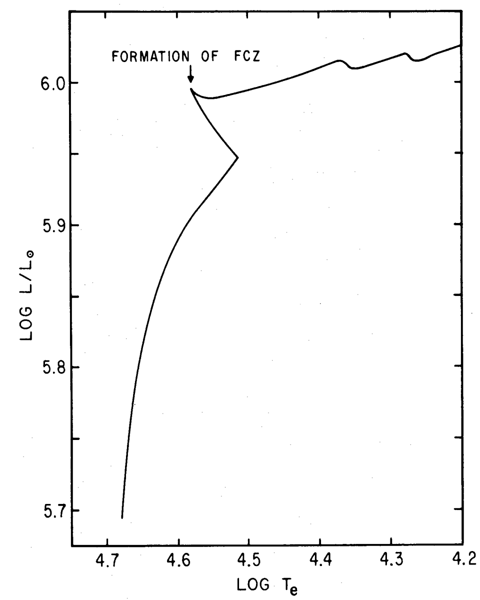

FIG. 4.—Theoretical H-R diagram showing a standard evolutionary track for a star of 60 $M_\odot$ from the zero-age main-sequence to the early post–main-sequence stages. Two pulse cycles of the thermally unstable hydrogen-burning shell (located at mass fraction $q_s = 0.38$) are detectable as surface luminosity fluctuations.

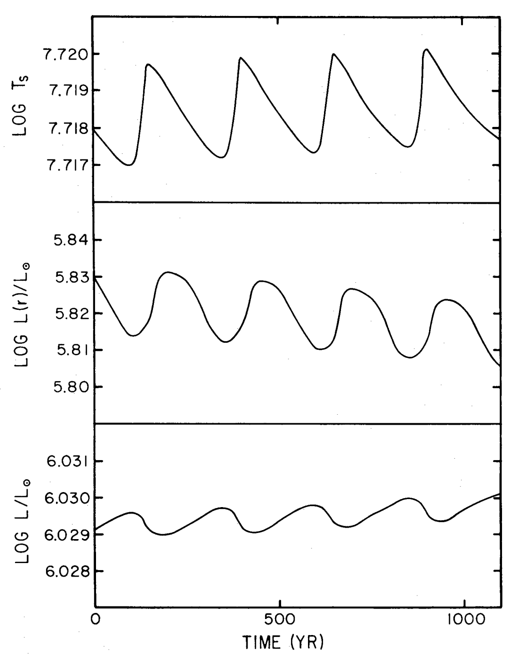

FIG. 3.—Four consecutive pulse cycles of the thermally unstable hydrogen-burning shell in a star of 60 $M_\odot$. The panels show, respectively, logarithms of the shell peak temperature, shell peak luminosity, and surface luminosity. The shell peak occurs at mass fraction $q_s = 0.35$.

c) Core Hydrogen Flash

Another possible source of thermal instability is a thermonuclear core flash, which can occur if a certain amount of hydrogen from the envelope is suddenly injected into the hot helium core after the original hydrogen is exhausted there (Stothers and Chin 1979, 1981). The difference between this core flash and the more familiar helium core flash in stars of low mass (e.g., Schwarzschild and Härm 1962) is that, in the massive stars, the gas in the core is nondegenerate even at the center, temperatures are very high to begin with, and the fuel (here hydrogen) must be supplied from outside the core.

In our previous work on this subject, we showed that, if the Schwarzschild criterion for convection is adopted and if the mass of the star is sufficiently high, the convective core may possibly connect and merge with the FCZ shortly after the FCZ is formed. On the assumption that a merger does take place, the mixing of these two convective regions leads to a radical chemical homogenization of a large fraction of the stellar interior. In dealing with the problem previously, we were content to skip the thermally unstable stages of evolution that immediately followed the merger, and we simply com-

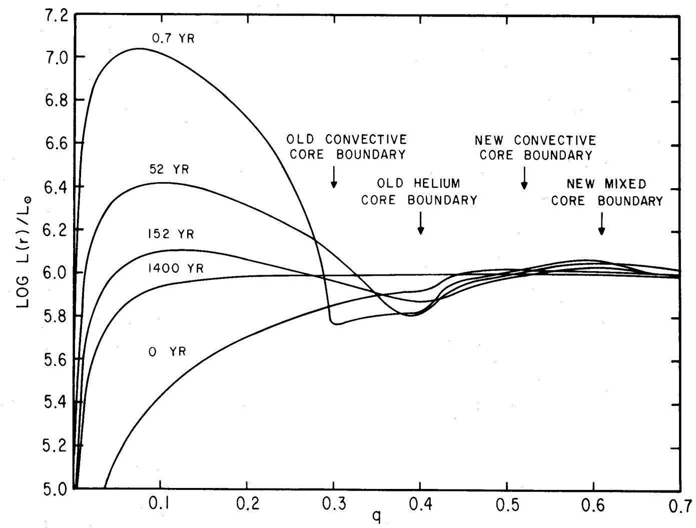

FIG. 5.—Luminosity profiles through the inner 70% of the mass of a star of 60 $M_\odot$, at various stages beginning at and following the merger between the FCZ and convective core.

puted the next model that was in full hydrostatic and thermal equilibrium under renewed central hydrogen burning. Here we shall compute the thermally unstable stages explicitly.

In order to obtain as strong a thermal flash as possible, we shall assume that the merger between the two convective zones takes place in the least time that can be justified physically, say, of the order of several convective overturning times. According to standard mixing-length theory, one convective overturning time in the core of a star of 60 $M_\odot$ is ~0.1 yr. We shall therefore specify, rather arbitrarily, a time step for the full merger equal to 0.7 yr, during which time the hydrogen content of the FCZ may be assumed to mix homogeneously over all mass layers with $q < 0.61$ (representing the outer boundary of the FCZ). Hydrodynamical effects can be neglected in these computations because the core's dynamical response time, $\tau_{\rm dyn} \approx (G\rho_c)^{-1/2}$, is only ~1 hr.

Before the merger, the only source of energy active in the hydrogen-exhausted core is potential energy of the changing gravitational field, which is the main supplier of heat to the surrounding hydrogen-burning shell. The run of luminosity through the star at this time is shown in Figure 5. During the merger, the hydrogen content in the core shoots up to $X = 0.12$. The high temperatures already prevailing there immediately trigger an almost explosive burning of the injected hydrogen, with a large release of energy that quickly begins to expand and cool the entire core. The attainment of a new core equilibrium configuration requires, according to Figure 5, about 1400 yr. This number agrees closely with the thermal response time calculated from the conventional expression

$$\tau_{\rm thermal} \approx \frac{6-3\beta}{2\beta}\,\frac{k\langle T\rangle}{\mu m_p}\,\frac{M_{\rm core}}{L_{\rm core}},$$

where β is the ratio of gas pressure to total pressure. The initial reaction of the surface of the star to the hydrogen core flash is a prompt, but very small, hydrostatic adjustment, amounting to $\delta M_{\rm bol} = 0.004$ mag and $\delta \log T_e = -0.009$. Then a rather slow decline in luminosity and effective temperature follows, as shown in Figure 6. The total drop in brightness attains 0.05 mag, which requires ~400 yr. Afterwards, the luminosity begins to climb again to a final equilibrium value of $\log(L/L_\odot) = 5.984$, which is associated with an equilibrium effective temperature of $\log T_e = 4.481$. When hydrogen eventually becomes exhausted in the core for the second time, a small FCZ develops again, but it never comes close enough to the convective core to

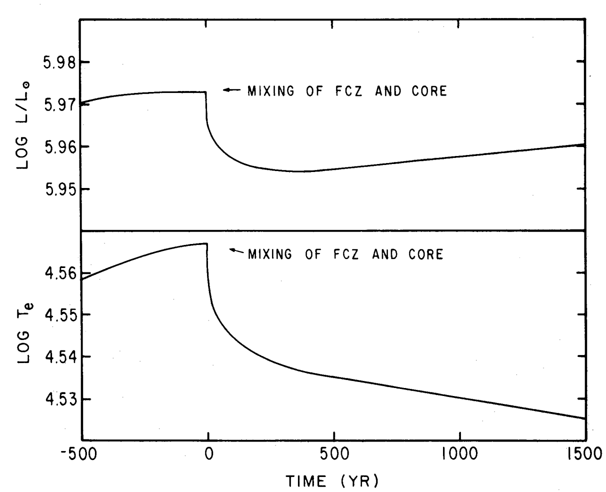

FIG. 6.—Evolution of the surface luminosity and effective temperature in a star of 60 $M_\odot$, at stages immediately preceding and following the merger between the FCZ and convective core.

produce another merger. An H-R diagram showing these phases has been published as Figure 2 in Stothers and Chin (1981).

In view of the very unremarkable changes in surface luminosity and effective temperature that have been calculated and of the lack of repetition of these changes, this mechanism for explaining the Hubble-Sandage variables must be rejected. Although some recurrence of core flashing could be achieved by supposing that a small amount of hydrogen becomes injected quasi-periodically into the center by a much milder form of overshooting (cf. Gabriel 1970), the resulting thermal pulses would necessarily be of lesser amplitude and therefore would still be of no practical interest here.

IV. PULSATIONAL INSTABILITIES

a) *Envelope-Energized Pulsations*

If the slow light cycles of 3–10 yr displayed by the Hubble-Sandage variables are due to a simple radial pulsation, these stars must be very near the limit of dynamical stability. By using linear adiabatic pulsation theory (Ledoux and Walraven 1958), we have computed the fundamental pulsation periods for several post–main-sequence models selected from our evolutionary tracks for a star of initially 60 $M_{\odot}$. The calculations were actually performed only for the outer 1% of the stellar mass, because the pulsational amplitudes become negligibly small at deeper layers.

According to our present evolutionary tracks, the effects of chemical evolution and mass loss produce a modest range of possible luminosities that the star may have, $\log(L/L_{\odot}) = 6.00$–6.13. Our most extreme model exhibits $\log(L/L_{\odot}) = 6.13$, which is associated with a greatly reduced mass $M/M_{\odot} = 30$ and a low surface hydrogen abundance $X = 0.220$. To achieve some generality, we shall allow the luminosity in the pulsation calculations to range from 1×10^6 $L_{\odot}$ up to the Eddington (1921) limit,

$$L_{\mathrm{E}} = 4\pi cGM/\kappa ,$$

where the force of radiation acting on matter with opacity κ just balances gravity. The results are shown in Figure 7 for two families of supergiant models, representing stars before extreme mass loss has occurred and the same stars afterwards; the two effective temperatures adopted are meant only to be representative.

Several interesting conclusions can be drawn from this figure. First, to obtain a really long pulsation period, the luminosity must evidently lie within about 2% of a somewhat variable upper limit. For hot stellar envelopes with no significant convection, this upper limit is just the Eddington luminosity; for cooler envelopes, it is the luminosity at which the partial-ionization zones of hydrogen and helium begin to occupy a large fraction of

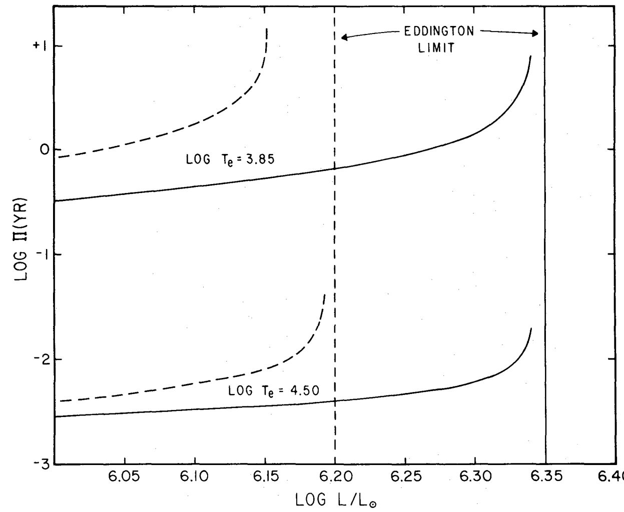

FIG. 7.—Period-luminosity relations for models of stellar envelopes pulsating in the fundamental radial mode. The models are characterized by $M/M_{\odot} = 60$, $X = 0.739$ (*solid lines*) and $M/M_{\odot} = 30$, $X = 0.220$ (*dashed lines*). The corresponding Eddington luminosity limits are indicated by vertical lines.

the fully pulsating layers. The sudden increase of the pulsation period actually follows fairly well from the approximate expression (Ledoux and Walraven 1958)

$$\Pi \approx \left[(3\langle\Gamma_1\rangle - 4)G\langle\rho\rangle\right]^{-1/2},$$

where Γ_1 is the generalized adiabatic exponent. Second, if we confine ourselves to $\log T_e > 3.85$ (corresponding to spectral types F and earlier, which are characteristic of most of the Hubble-Sandage variables), the longest pulsation period occurring for models of 60 $M_{\odot}$ that actually lie on the present evolutionary tracks is only a few months; but for the reduced-mass models of 30 $M_{\odot}$, it reaches 3 yr, which is close enough to the observed periods to be significant.

Since the present tracks by no means cover all possible evolutionary histories, we ought to consider at least one physically realistic alternative history that can lead to an even longer period. In a previous paper (Stothers and Chin 1981) we showed that if the star undergoes a prior (but brief) red-supergiant phase, deep envelope convection will mix hydrogen from the outer layers down to the helium-core boundary. Consequently, after mass loss takes place, the star will contain not only a small mass M but also a high (almost normal) hydrogen abundance at the surface X. The combination of these two factors reduces the Eddington luminosity, since the opacity is proportional to $1 + X$. On the other hand, the star's actual luminosity remains essentially unchanged, because it is generated almost entirely by the helium core. Therefore the pulsation period will increase. To determine the stellar luminosities for which the increase of period becomes very large, we shall consider a sequence of supergiant models consisting of a massive helium-burning core surrounded by a small hydrogen-rich envelope, whose mass we will, for simplicity, take to be negligible, although to produce a real supergiant a few percent of the star's total mass should be placed in the envelope. Arnett's (1972) helium-star models covering the mass range 16–100 $M_\odot$ provide the needed mass-luminosity relation:

$$\log(L/L_\odot) = 3.12 + 2.37\log(M/M_\odot) - 0.30[\log(M/M_\odot)]^2.$$

Evolutionary effects in the models have been computed both by Arnett (1972) and by Deinzer and Salpeter (1964), who find that, when the central helium abundance has fallen to $Y_c \approx 0.02$, the models are brighter by

$$\delta \log(L/L_\odot) = 0.23 - 0.085\log(M/M_\odot).$$

Figure 8 displays the mass-luminosity relations for the two cases $Y_c \approx 1$ and $Y_c \approx 0.02$. If the luminosity of the star is to equal or exceed the Eddington limit, its envelope hydrogen abundance X must be equal to or greater than the corresponding value indicated, which has been computed by assuming a purely electron-scattering opacity, $\kappa = 0.2(1+X)$ cm^2 g^{-1}. Since, in practice, $X \leq 0.7$, the Eddington limit can be equaled or

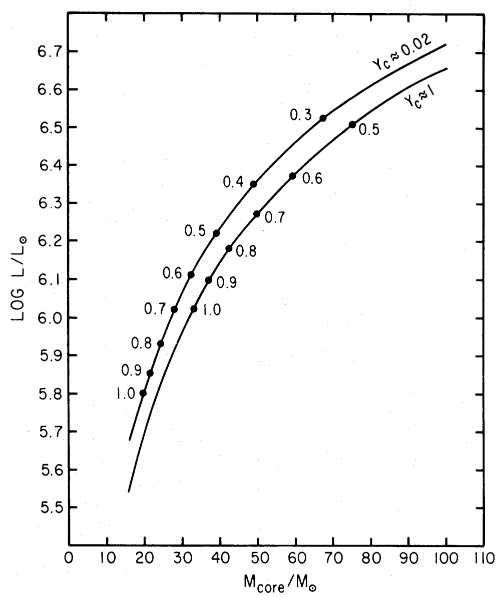

FIG. 8.—Mass-luminosity relations for models of stars with massive helium cores and hydrogen-rich envelopes of negligible mass. Results for two values of the central helium abundance Y_c are shown. The attached series of numbers refer to the minimum envelope hydrogen abundance X that is necessary to have the star's luminosity exceed the Eddington limit.

surpassed only if $\log(L/L_\odot) \geq 6.0$. Interestingly, this is just the region in which the Hubble-Sandage variables are observed.

A stellar situation in which the luminosity significantly exceeds the Eddington limit cannot persist for long, because the excess radiation pressure will drive off matter on a rapid, dynamical time scale (e.g., Sparks, Starrfield, and Truran 1978),

$$\tau_{\rm dyn} \approx \left[\left(\frac{L}{L_{\rm E}} - 1\right)\frac{GM}{R^3}\right]^{-1/2}.$$

This process will at first feed on itself owing to the fact that a reduced stellar mass lowers $L_{\rm E}$. Eventually, however, the increasingly hydrogen-poor character of the surface will lower the opacity sufficiently for $L_{\rm E}$ to become equal to L. Thereafter, mass loss will still continue, but only very slowly, for the reason that a gradual increase of L will be caused by evolution of the helium core. We suggest that this ongoing mass loss will manifest itself as random outbursts (in analogy with novae), which impulsively start up oscillations of the envelope for a few pulsation periods.

b) *Pulsations of a Circumstellar Shell*

There is no *a priori* reason why an optically thick circumstellar gas or dust shell could not pulsate. The period would be extremely long if the shell radius were sufficiently large. One argument against applying this idea to the Hubble-Sandage variables, however, is that the shell is probably not optically thick enough in many of these objects to be self-excited by the mechanisms usual in cool stellar envelopes, although mass flow from the underlying star might impulsively drive pulsations. Another problem is that, despite a marked disparity between the optical thicknesses of the circumstellar shells observed around different Hubble-Sandage variables, the "periods" are very similar. For example, the relatively mildly obscured S Dor shows a cycle of ~10 yr (Gaposchkin 1943; van Genderen 1979), while the heavily obscured η Car has one of ~15 yr (Payne-Gaposchkin 1957) or possibly ~3 yr (Feinstein and Marraco 1974). Two intermediate cases, Variable 19 (AF And) in M31 and Variable B in M33, show intermittent cycles of ~4 yr and ~7 yr, respectively (Hubble and Sandage 1953; Rosino and Bianchini 1973). Nevertheless, the theoretical possibility of a pulsating circumstellar gas or dust shell is perhaps worth examining in future work.

c) *Giant Convection Cells*

The overturning of giant convection cells (nonradial g^- modes) in turbulent stellar envelopes is a potentially visible form of nonradial stellar oscillations. In the atmospheres of red supergiants these cells possibly cause the secondary light and velocity variations that are observed with periods of ~ 10 yr (Stothers and Leung 1971), but in bluer supergiants the giant cells (de Jager and Vermue 1979; de Jager 1980) as well as the nonradial g^+ modes (Lucy 1976; Maeder 1980) are far less conspicuous; therefore, there is little likelihood that they can explain the Hubble-Sandage variables.

d) *Core-Energized Pulsations*

Very massive stars on the zero-age main sequence (ZAMS) are, according to well-founded theory, unstable to pulsations excited by nuclear reactions in the core (Ledoux 1941; Schwarzschild and Härm 1959). Much theoretical work has led to the following observational predictions for these stars. First, substantial driving should appear only in the fundamental radial mode and not in the radial overtones (Simon and Stothers 1969*b*; Ziebarth 1970; Papaloizou 1973*a*) or in the nonradial modes (Wan 1966; Aizenman, Hansen, and Ross 1975); therefore the expected pulsation periods are of the order of several hours. Second, evolution off the ZAMS ought to rapidly quench the pulsations (Schwarzschild and Härm 1959; Stothers and Simon 1968; Simon and Stothers 1969*a*, 1970; Van der Borght 1969), so that the minimum mass for pulsational instability ought to be simply the ZAMS critical mass for nonrotating, nonmagnetic stars, $\sim 80\ M_\odot$, which corresponds to a luminosity of $\sim 1 \times 10^6\ L_\odot$ (Stothers and Simon 1970; Ziebarth 1970). Third, the unstable models do not actually disrupt themselves but appear to limit their amplitudes by means of shock wave dissipation near the surface (Appenzeller 1970*a*, *b*; Ziebarth 1970; Talbot 1971; Papaloizou 1973*b*); thus they become enlarged by up to 4 times in mean photospheric radius, and, every few periods, as the atmospheric shock waves build up in strength, they may eject an optically thick shell of material which temporarily obscures the star and may not respond noticeably, or else may respond with a longer period, to the underlying pulsations (see also Ungar 1971). A rapid enough succession of shells would of course keep the photosphere continually obscured. But since the predicted rates of mass loss depend so sensitively on the uncertain shock strength, estimates of the mass loss range from no mass loss at all (Ziebarth 1970; Papaloizou 1973*b*) up to an energy-limited value which, however, is only $4 \times 10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$ (Appenzeller 1970*a*) even for a mass as high as $130\ M_\odot$.

S Dor is the most extensively observed Hubble-Sandage variable. However, it shows no periodic light variability on a time scale of several hours to a day (Mendoza 1970; Appenzeller 1974), and its measured rate of mass loss, $5 \times 10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$ (Stahl and Wolf 1982), may be too large for a nuclear-energized pulsator. On the other hand, one could assume that nuclear-energized pulsations and normal stellar-wind mechanisms combine, in stars of exceptionally high mass, to produce massive opaque flows that obscure the photosphere. Further modeling of these possible flows would be worthwhile, although S Dor itself does not appear to be excessively obscured.

V. CONCLUSION

A list of proposed mechanisms for the slow variations of the Hubble-Sandage (or S Doradus) variables has been assembled in this paper. These mechanisms include (see § I for additional suggested mechanisms)

1. Episodic, nearly catastrophic mass loss, perhaps accompanied by temporary shrinkage of the stellar radius, in a luminous supergiant.
2. Flickering of a hydrogen-burning or helium-burning shell.
3. Hydrogen flashing in an evolved stellar core.
4. Pulsation of a luminous supergiant envelope near the Eddington limit of radiative stability.
5. Pulsation of a dense circumstellar gas or dust cloud.
6. Overturning of giant convection cells (or some other kind of nonradial oscillation) in a luminous supergiant envelope.
7. Vibrational instability and mass outflow in an extremely massive main-sequence star.

Among these various mechanisms, we have been able to rule out, fairly conclusively, the second and third ones listed above. The sixth and seventh ones are also unlikely. Our strong preference is for the first mechanism, which seems to explain well the large and sudden declines in blue light that are followed by very slow recoveries. Our version of the first mechanism, which is relevant when the ejected cloud is not too optically thick, and Davidson's (1971) version, which is based on a very optically thick cloud, have been applied here to different subgroups of the observed variables. In both cases, our theoretical models for the underlying star predict that the star will be undermassive for its luminosity by a factor of ~ 2, helium and nitrogen will be overabundant at its surface, and its bolometric light output will remain essentially constant during and after the massive outbursts.

Perhaps a more difficult problem in interpreting the Hubble-Sandage variables is posed by the observation of large-amplitude blue-light oscillations with "periods" of 3–10 yr. If these variations are not simply due to rapidly recurring episodes of mass loss, then they may be indicators of bulk pulsations either of the stellar envelope (if the star's luminosity lies close to the Eddington limit) or else of the circumstellar cloud (if the cloud is optically thick enough). Possibly the pulsations are being driven by the outward mass flux from the star (or by whatever causes the outward mass flux). Although one cannot entirely exclude the possibility that several different mechanisms may be operating either simultaneously or individually in different Hubble-Sandage variables, the similarly bright luminosities of these variables do suggest that they comprise a homogeneous class of objects. It is clear that much more work—both theoretical and observational—remains to be done on these curious and important variable stars.

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